Single-step implementation of the controlled-Z gate in a qubit/bus/qubit device

Andrei Galiautdinov
Department of Electrical Engineering, University of California, Riverside, California 92521, USA
(Dated: January 16, 2013)

We propose a simple scheme for generating a high-fidelity controlled-Z (CZ) gate in a three-component qubit/bus/qubit device. The corresponding tune/detune pulse is *single-step*, with a near-resonant constant undershoot between the $|200\rangle$ and $|101\rangle$ states. During the pulse, the frequency of the first qubit is kept fixed, while the frequency of the second qubit is varied in such a way as to bring the $|200\rangle$ and $|101\rangle$ states close to resonance. As a result, the phase of the $|101\rangle$ state is accumulated via the corresponding *second-order* anticrossing. For experimentally realistic qubit frequencies and a 75 MHz coupling (150 MHz splitting), a 45 ns gate time can be realized with $> 99.99\%$ intrinsic fidelity, with errors arising due to the non-adiabaticity of the ramps. The CZ pulse is characterized by *two* adjustable parameters: the undershoot magnitude and undershoot duration. The pulse does *not* load an excitation into the bus. This by-passes the previously proposed need for two additional qubit-to-bus and bus-to-qubit MOVE operations. Combined with the recently predicted high-fidelity idling operation in the RezQu architecture [A. Galiautdinov, J. Martinis, A. Korotkov (unpublished)], this controlled-Z scheme may prove useful for implementations on the first generation quantum computers.

PACS numbers: 03.67.Lx, 85.25.-j

I. INTRODUCTION

Recent progress in preparing, controlling, and measuring the macroscopic quantum states of superconducting circuits with Josephson junctions [1–7] makes realization of a quantum computer an experimental possibility [8]. Two major roadblocks – decoherence and scalability – may soon be overcome by the so-called Resonator/zero-Qubit (RezQu) architecture, recently proposed by J. Martinis [9]. Some of the basic operations of the RezQu architecture (such as the idling operation, the generation and measurement of the single-excitation states, as well as the single-excitation transfer operation called MOVE) were analyzed in a joint paper [10]. It was found, that the RezQu architecture is capable of providing high-fidelity performance required for quantum information processing.

In spite of the optimistic conclusions presented in Ref. [10], an important problem of generating *high-fidelity entangling* operations in the RezQu architecture still remains. One such operation is the controlled-Z (CZ) gate, given in Eq. (8). It is believed that, in the RezQu architecture, the CZ gate may easily be produced using the SWAP-based *three-step* approach, similar to that of Ref. [12], in which one logic qubit is moved to the bus, transferring the qubit excitation onto the bus, while the other qubit is tuned close to resonance with the bus for a precise duration. After the needed phase is accumulated (as in Refs. [13, 14]), the excitation is moved back from the bus to the original qubit. We have simulated this three-step approach for realistic RezQu parameters and found some difficulties with it, which are described in Section III. This prompted us to look for a more direct scheme, which is not beset by such difficulties. The scheme is described in Sec. V. It does not rely on the loading and unloading of the bus, but instead, uses a second order anticrossing for the required phase accumulation (*cf.* Ref. [15]).

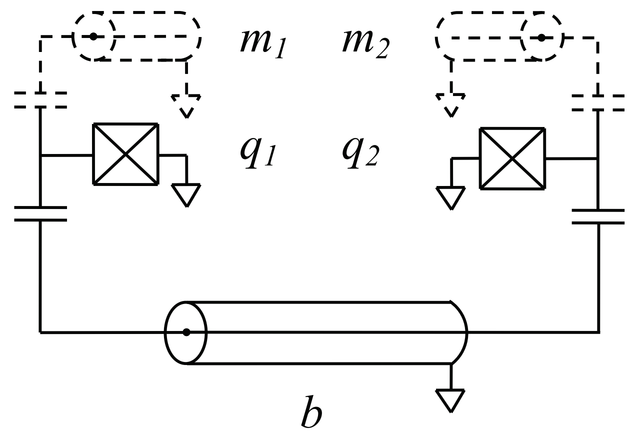

FIG. 1: Schematic diagram of a qubit/bus/qubit RezQu device. The qubits may be supplemented with memory resonators (dashed lines). Here, q – qubits, b – bus, m – memory resonators.

II. THE QUBIT/BUS/QUBIT DEVICE

A three-component RezQu device is depicted in Fig. 1. In the rotating wave approximation (RWA), its dynamics is described by the Hamiltonian

$$H(t) = \sum_{i=1,2} H_i(t) + \omega_b a_b^\dagger a_b + g_{b1}\left(\sigma_1^- a_b^\dagger + \sigma_1^+ a_b\right) + g_{b2}\left(a_b^\dagger \sigma_2^- + a_b \sigma_2^+\right), \quad (1)$$

TABLE I: Configuration of the $(q_1 b q_2)$ system before, after, and during the CZ gate. All frequencies are defined by $\nu = \omega/2\pi$ (GHz) and are *bare*. Both qubit anharmonicities are $\eta/2\pi = 0.2$ GHz and assumed to be constant. The coupling is chosen to be $g_b/2\pi = 75$ MHz to guarantee sufficiently fast, $t_{\text{gate}} = 45$ ns, gate operation. Left arrows indicate the near-resonant two-excitation frequencies.

Bare frequencies	Before and after CZ	Optimized CZ frequencies at the 200 $\leftrightarrow$ 101 anticrossing
ν_{q1}	6.6	6.6
ν_b	6.0	6.0
ν_{q2}	6.5	6.40959
ν_{110}	12.6	12.6
ν_{101}	13.1	13.00959 $\leftarrow$
ν_{011}	12.5	12.40959
ν_{200}	13.0	13.0 $\leftarrow$
ν_{020}	12.0	12.0
ν_{002}	12.8	12.61918

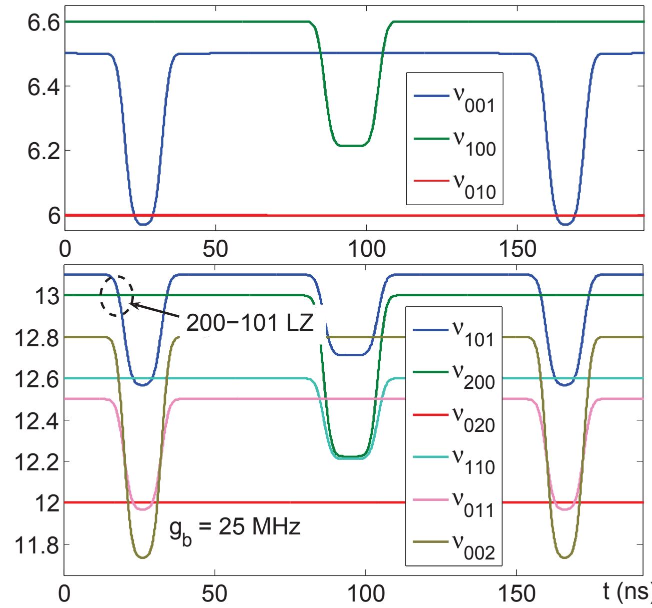

FIG. 2: (Color online) One- and two-excitation frequencies (in GHz) of the qubit/bus/qubit system in the usual 3-step SWAP-based CZ gate implementation. Optimization with at least *four* naturally chosen parameters at $g_b/2\pi = 25$ MHz gives the gate error, as defined in Eq. (9), of no less than 2×10^{-3}. Compare with Fig. 3, where an alternative, single-step CZ-generating scheme is presented.

where

$$H_i(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \omega_i(t) & 0 \\ 0 & 0 & 2\omega_i(t) - \eta_i \end{bmatrix} \qquad (2)$$

are the Hamiltonians of the qubits whose frequencies ω_i may vary in time and the anharmonicities η_i are assumed to be constant,

$$\sigma_i^- = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{bmatrix}, \quad \sigma_i^+ = \left(\sigma_i^-\right)^\dagger, \qquad (3)$$

are the qubit lowering and raising operators, ω_b is the bus frequency (which is held fixed), $a_b^\dagger$ and a_b are the creation and annihilation operators for the bus photons, and g_{b1}, g_{b2} are the bus-qubit coupling constants. In our numerical simulations we will assume that $\eta_1 = \eta_2 \equiv \eta$ and $g_{b1} = g_{b2} \equiv g_b$.

III. SOME DIFFICULTIES WITH THE SWAP-BASED CONTROLLED-Z GATE IMPLEMENTATION

In what follows, we assume that the qubit/bus/qubit $(q_1 b q_2)$ system starts and ends in the (default) configuration, as shown in Table I. The initial and final qubit frequencies, $\omega_{q1} > \omega_{q2}$, are chosen in such a way as to avoid the $|100\rangle \leftrightarrow |001\rangle$ and $|200\rangle \leftrightarrow |101\rangle$ crossings. Then, the standard [14] three-step SWAP-based CZ gate implementation (Fig. 2) suffers from the following major drawback: it produces a large number of Landau-Zener (LZ) transitions, each of which degrades the resulting gate's fidelity. The RezQu architecture based on fixed couplings may not be flexible enough to provide the needed controllability to counter the effects of all these transitions in a three-step CZ manner. Controlling only the qubit frequencies may not be enough to achieve the needed 10^{-4} accuracy of the CZ gate, using *experimentally reasonable number* of parameters. As Fig. 2 shows, the very first ramp of the initial SWAP operation already contains one such LZ crossing, leading to the leakage from state $|101\rangle$ to state $|200\rangle$.

Here we propose to turn this particular drawback into an asset by dropping the loading and unloading SWAP operations altogether and employing the mentioned $|200\rangle \leftrightarrow |101\rangle$ anticrossing to accumulate the needed 101-phase during the CZ operation (see Fig. 3).

IV. POTENTIAL PROBLEMS WITH THE PROPOSED SCHEME

The following two problems may arise in our scheme.

First, the presence of additional system elements (qubits, memory resonators, etc.) may result in additional states that are near-resonant with the states $|200\ldots\rangle$ and $|101\ldots\rangle$, thus leading to unwanted leakage. Here we ignore this complication and assume that under realistic conditions it will always be possible to isolate this particular anticrossing sufficiently well.

Second, being a second-order process, accumulation of the 101-phase may proceed too slowly compared with the qubit coherence time (currently at about $t_{\text{coherence}} \simeq 500$ ns). However, the following argument shows that this is

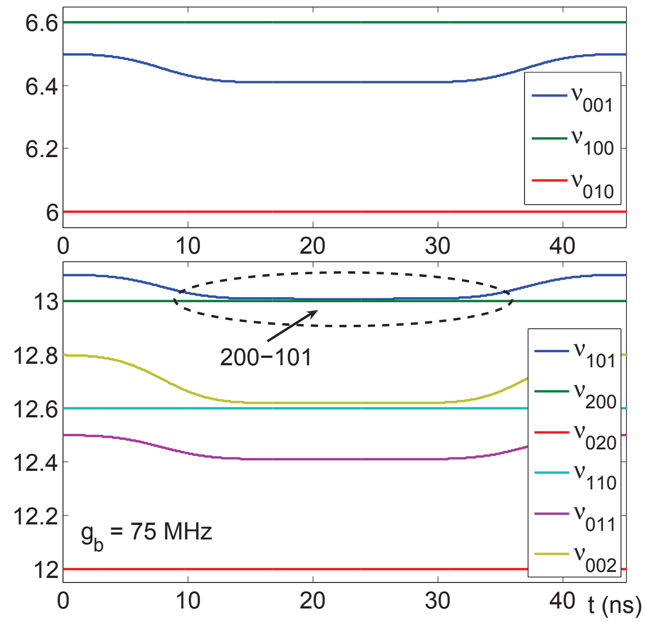

FIG. 3: (Color online) One- and two-excitation frequencies (in GHz) of the qubit/bus/qubit system in the single-step CZ gate implementation. A two-parameter optimization at $g_b/2\pi = 75$ MHz gives $> 99.99\%$ fidelity for total gate duration of $t_{\rm gate} = 45$ ns. The optimized parameters are the undershoot, $(\omega_{101} - \omega_{200})/2\pi = 9.59$ MHz, and the undershoot duration, $t_{\rm undershoot} = 29.1$ ns (measured between the central points of the two ramps). The widths of the error-function-shaped ramps were held fixed at $\sigma_{\rm in} = \sigma_{\rm fin} = 3$ ns. Compare with Fig. 2.

not necessarily true.

In the case of a similar second order resonance $|100\rangle \leftrightarrow |001\rangle$, the effective (via the bus) q_1-q_2 coupling [11] is given by

$$g_{\rm eff}^{100\leftrightarrow 001} = \frac{2g_b^2\omega_b}{\omega_{q1}^2 - \omega_b^2}. \tag{4}$$

Then in our case we should have

$$g_{\rm eff}^{200\leftrightarrow 101} = \sqrt{2} g_{\rm eff}^{100\leftrightarrow 001}. \tag{5}$$

Choosing $g_b = 75$ MHz (experimentally achievable coupling) and setting $\omega_{q1}/2\pi \approx 6.4$ GHz (near-resonant condition), we find for $\omega_b/2\pi = 6.0$ GHz,

$$\begin{aligned} \frac{g_{\rm eff}^{200\leftrightarrow 101}}{2\pi} &= \sqrt{2}\left(\frac{2\times 0.075^2 \times 6.0}{6.4^2 - 6.0^2}\right) \\ &\approx 0.0192\ \text{GHz} = 19.2\ \text{MHz}, \end{aligned} \tag{6}$$

which gives an experimentally reasonable duration of the corresponding phase accumulation,

$$t_{2\pi\ \rm pulse}^{200\leftrightarrow 101} = \frac{\pi}{g_{\rm eff}^{200\leftrightarrow 101}} \approx 26\ \text{ns}. \tag{7}$$

In the actual implementation shown in Fig. 2, the total gate duration had to be prolonged to $t_{\rm gate} = 45$ ns in order to correctly produce the final populations of $|100\rangle$ and $|001\rangle$ states.

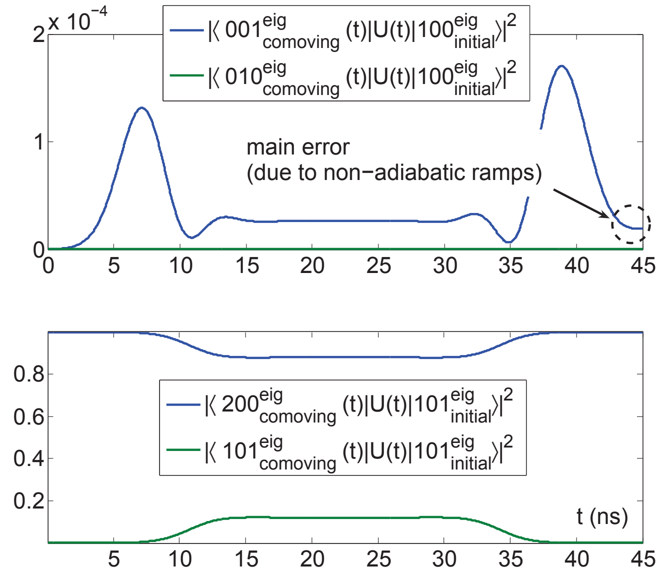

FIG. 4: (Color online) Some of the overlaps between the time-evolving computational states and the comoving system eigenstates in the single-step CZ gate implementation. $U(t)$ stands for the unitary operator of the time evolution up to time t.

V. IMPLEMENTING THE SINGLE-STEP CZ GATE

The proposed single-step CZ gate resulting from a two-parameter optimization with the RWA Hamiltonian of Eq. (1) is depicted in Fig. 3. To provide some intuitive understanding of how the generated gate works, the overlaps between the time-evolved logic states and some of the time-dependent (comoving) system eigenstates are given in Fig. 4.

Our gate is implemented in the computational basis consisting of the full system eigenstates [10]. It has the form

$$\text{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\varphi_1} & 0 & 0 \\ 0 & 0 & e^{-i\varphi_2} & 0 \\ 0 & 0 & 0 & -e^{-i(\varphi_1+\varphi_2)} \end{pmatrix}, \tag{8}$$

where φ_1 and φ_2 are some arbitrarily accumulated phases. Due to the use of the system *eigenstates*, these phases can always be adjusted simply by waiting. The optimization was performed at fixed $t_{\rm gate} = 45$ ns by minimizing the function

$$\begin{aligned} \text{Error}(U) &= \text{Error}_1 + \text{Error}_2 + \text{Error}_3 + \text{Error}_4 \\ &= \left(1 - |a_1|^2\right) + \left(1 - |a_2|^2\right) + \left(1 - |a_3|^2\right) \\ &\quad + \left|1 + \frac{a_1 a_2 a_3^*}{|a_1 a_2 a_3^*|}\right| \geq 0, \end{aligned} \tag{9}$$

where

$$a_1 = \langle\overline{100}|U|\overline{100}\rangle, a_2 = \langle\overline{001}|U|\overline{001}\rangle, a_3 = \langle\overline{101}|U|\overline{101}\rangle, \tag{10}$$

with respect to the undershoot magnitude and the undershoot duration (measured between the central points of the ramps), with additional constraints $\mathrm{Error}_1 + \mathrm{Error}_2 + \mathrm{Error}_3 < 10^{-4}$ and $\mathrm{Error}_4 < 10^{-10}$. The widths (standard deviations) of the error-function-shaped ramps were held fixed at 3 ns. The results are presented in Fig. 3. In the above, U is the unitary operator representing the CZ pulse, and the overbars stand for the prefix "eigen-." The optimization function $\mathrm{Error}(U)$ was defined so that for $U = \mathrm{CZ}$, as given in Eq. (8), $\mathrm{Error(CZ)} = 0$. Notice, that we do not have to take into account the phase of the $|000\rangle$ state, since the corresponding frequency ν_{000} can always be set to 0.

VI. CZ GATE AS AN IDLING ERROR

Our CZ gate may be viewed as a particular example of an "idling error" [10], which is a measure of how fast the phase of the *computational* eigenstatestate $|\overline{101}\rangle$ accumulates relative to the phases of eigenstates $|\overline{100}\rangle$ and $|\overline{001}\rangle$. The error is characterized by the running frequency $\Omega_{ZZ} = \varepsilon_{101} - \varepsilon_{100} - \varepsilon_{001} + \varepsilon_{000}$, with ε_{ijk} being the corresponding eigenenergies, and physically arises due to the level repulsion between 101 and other levels in the two-excitation subspace of the system. Consequently, a superposition of computational states evolves as

$$\begin{aligned} |\psi(t)\rangle \;=\; & \alpha_{000} e^{-i\epsilon_{000}t}|\overline{000}\rangle + \alpha_{100} e^{-i\epsilon_{100}t}|\overline{100}\rangle \\ & + \alpha_{001} e^{-i\epsilon_{001}t}|\overline{001}\rangle \\ & + \alpha_{101} e^{-i\Omega_{ZZ}t} e^{-i(\epsilon_{100}+\epsilon_{001}-\epsilon_{000})t}|\overline{101}\rangle, \end{aligned} \tag{11}$$

and so, after a time $t_{\mathrm{cp}} = \pi/\Omega_{ZZ}$, the state $|\overline{101}\rangle$ gets multiplied by -1. Thus, in systems with nonlinearities, the CZ gate can always be generated simply by waiting. For the qubit/bus/qubit RezQu device, using Eq. (1), we find in fourth order,

$$\Omega_{ZZ}^{(4)} = \frac{2g_{b1}^2 g_{b2}^2 \left\{\omega_1\eta_1(2\omega_b - \omega_1 - \eta_2) + \omega_2\eta_2(2\omega_b - \omega_2 - \eta_1) - \omega_b\left[\omega_b(\eta_1+\eta_2) - 2\eta_1\eta_2\right]\right\}}{(\omega_1-\omega_b)^2(\omega_2-\omega_b)^2\left[\omega_1 - (\omega_2-\eta_2)\right]\left[(\omega_1-\eta_1)-\omega_2\right]}, \tag{12}$$

which for the above mentioned (and fixed) $\omega_1/2\pi = 6.6$ GHz and $\omega_2/2\pi = 6.5$ GHz, at $g_b/2\pi = 75$ coupling, produces the CZ gate after about 130 ns, which is too long. For a more efficient CZ generation, the system parameters must be set to maximize Ω_{ZZ}. One such choice, $\omega_2 \approx \omega_1 - \eta_1$, which corresponds to the $200 - 101$ anticrossing, was made in our single-step implementation.

VII. CONCLUSION

To summarize, we introduced a scheme for single-step generation of a high-fidelity controlled-Z gate in a three-component RezQu architecture. Despite the use of a second-order anticrossing, the accumulation of the needed 101-phase proceed sufficiently fast compared with the qubit coherence time. Different from the usually considered proposals, our CZ scheme does not rely on the MOVE operations transferring excitations to and from the bus. The resulting simplicity of the generated gate may prove useful for implementations in the first generation solid state quantum computers.

Acknowledgments

This work was supported by NSA/IARPA/ARO Grant No. W911NF-10-1-0334.

[1] Y. Yamamoto et al, Nature **425**, 941 (2003).
[2] M. Steffen et al., Science **313**, 1423 (2006).
[3] J. H. Plantenberg et al., Nature **447**, 836 (2007).
[4] M. Neeley et al., Nature Physics **4**, 523 (2008).
[5] L. DiCarlo et al, Nature **460**, 240 (2009).
[6] R. C. Bialczak et al., Nature Physics **6**, 409 (2010).
[7] M. D. Reed et al., Phys. Rev. Lett. 105, 173601 (2010).
[8] J. Clarke and F. K. Wilhelm, Nature **453**, 1031 (2008).
[9] J. M. Martinis (unpublished).
[10] A. Galiautdinov, J. Martinis, A. Korotkov, presented at the 2011 APS March Meeting, Dallas, TX (unpublished).
[11] R. A. Pinto et al., Phys. Rev. B **82**, 104522 (2010).
[12] G. Haack et al., Phys. Rev. B **82**, 024514 (2010).
[13] F. W. Strauch et al., Phys. Rev. Lett. **91**, 167005 (2003).
[14] T. Yamamoto et al., Phys. Rev. B **82**, 184515 (2010).
[15] Shi-Biao Zheng, Appl. Phys. Lett. **94**, 154101 (2009).